## W hy Displacement M atters: A Green/ Purple Debate

## Game Scale And B asic Equations

1 Hex $=10 \mathrm{~km}, 1$ turn $=64$ seconds. (DV scale, VM S/FT have no specified scale)
Velocity $=$ a*t, displacement $=0.5 * a * t{ }^{2}$. A velocity change of 2 hexes/turn is 0.5 g of thrust.

## The Fundemental Problem:

The example to the right shows three vector plots for a multi-turn linear thrust from an at-rest state. Black circles show unit positions at the end of a constant acceleration. Gray circles show positions after the engines are turned off, with the unit dr ifting at its new velocity.
The example in the green box is thrust 2 in the Full Thrust-style Vector M ovement System (VM S). VM S assumes that thrust is either instantaneous, or that the dis placement on the first turn of thrust is equalto the velocity change.
The first purple box is thrust 1 in DV scale, where thrust 1 equates to one additional hex of movement (displacement) on the turn of thrust, and a velocity change of 2 hexes/turn. This matches the velocity change in the thrust 2 VM S example
The second purple box is thrust 2 in DV scale. This matches the turn 1 displacement of the thrust 2 VM S example, but has a greater discrepancy in velocity change. It is the only example where VM S and DV place a ship in the same hex at the same point in time.
Because it measures from a fixed point, the graphic at right understates the full impact of these discrepancies. Spaceships aren't fixed points, and velocity change and position are two parts of a tactic alenvironment.

The error increases linearly with the rate of thrust. Going to thrust 4 results in double the error above; two units thrusting towards each other at thrust 4 in VM S would have a combined positional error of 8 hexes after the first turn, which is significant in most games that can be played on a hex map, and thrust 4 in the context of the DV game scale is fairly modest. As the game progresses, the errors accumulate and multiply over time, and also tend to expand at the rate of $(n \wedge 2-n)^{t}$ where $n$ is the number of units on the map engaging in thrust and $t$ is the number of turns the game has progressed.


## Facing C hanges: A New Problem

At left is a blue bounding box that shows what happens when facing changes are allowed in the middle of a burn. Assuming the turn took place at the exact midpoint of the burn, there would be no displacement on the hex grid in proper Newtonian movement. M aking the vector change and displacement equalresults in a significant positionalerror when dealing with burns of less than one time increment. As the number of facing changes dur ing a burn increases, the rate of error increases similarly.
M ultiple S hip Example
The graphic below shows how each discrepancy is magnified by both the length of the game and the number of units in play. Each star represents the start ing point for a unit. Dark green/ purple circles indicate positions obtained via dis placement+velocity drift. W hen VM S and DV would place the unit in the same hex, a split green/ purple circle is used. The light er color labels indicate where a unit would be after drifting at velocity. In the illustration below,thrust 1 equat es to half a g of thrust, and assumes that VM S and DV are equivilant in dis placement, rather than in velocity change. Unit A has initial vectors of 4 in D, 2 in C and does not thrust on turn 1 , then thrusts for 4 in $F$ on turn 2 . Unit B starts at rest, thrusts at 7 in $C$ on turn 1, and thrusts at 4 in A on turn 2. Unit C has an initial vector of 3 in $E$, thrusts at 5 in B onturn 1, and stops thrust on turn 2 . All three ships drift on turn 3. Compare the discrepancies in expected positions of on turn 3. In particular, note the radically different positions of units $A$ and $B$. Rather than having a range 12 pass at the end of turn 2 (VMS), they actually had a range 4 shot in $D V$, while by turn 3 they've already passed each other with 16 hexes of separation, versus a range 11 shot in VMS.


