# ROCKET EQUATIONS

**m** = meter
**kg** = kilogram
**x^2** = square of **x**
**sqrt[x]** = square root of **x**
**g** = 9.81 m/sec^2
**1n[x]** = Log(*e*), natural logarithm of **x**
**G** = 6.67206e–11 Nm^2/kg^2
**c** = 299792458 m/sec
**$\pi$** = 3.141592654
**Va** = average velocity (*m/sec*)
**V** = change in velocity (*m/sec*)
**Vi** = initial velocity (*m/sec*)
**Vf** = final velocity (*m/sec*)
**S** = change in distance (*m*)
**T** = time (*seconds*)
**A** = acceleration (*m/sec^2*)
**Ai** = "instantaneous" acceleration (*m/sec^2*)

**ENGINE PARAMETERS**

**F** = Thrust (*Newtons* or *kg m/sec*)
**Pw** = Thrust Power (*kW*)
**Isp** = Specific Impulse (*seconds*)
**Mdot** = Propellant mass flow (*kg/sec*)
**Ve** = Velocity of exhaust (*m/sec*)
**Mps** = Mass of propulsion system (power plant+thrust system) (*kg*)
**dMp** = Mass of propellant burnt in current burn (*kg*)
**Mp** = Total mass of propellant carried (*kg*)
**Alpha** = Specific Power = Pw / Mps (*kW/kg*) = **$\alpha$**
**Vch** = Characteristic Velocity
**Epsilon** = percentage of propellant mass converted into energy = **$\varepsilon$**

**VEHICLE PARAMETERS**

**Mpl** = Mass of ship's payload (*kg*)
**Ms** = Ship's structural mass (*kg*)
**Mt** = Ship total mass = Mp + Mpl + Mps + Ms (*kg*)
**Me** = Ship's mass empty (i.e., all propellant burnt) (*kg*)
= Mt – Mp
**Mc** = Ship's "current" mass (at this moment in time) (*kg*)
**Mbs** = Ship's mass at start of current burn (*kg*)
{At start of mission = Mt. Later it is Mt – (sum of all DMp's of all burns)}
**Mbe** = Ship's mass at end of current burn (*kg*)
**Lambda** = Ship's mass ratio = Mt / Me = **$\lambda$**
**deltaV** = Ship's total velocity change capability (*m/sec*) = **$\Delta$V**
**dTm** = Maximum duration of burn (*seconds*)
**Gamma** = relativistic factor = **$\gamma$**

**MISSION PARAMETERS**

**deltaVb** = Velocity change of current burn (*m/sec*)
**dT** = Duration of current burn (*seconds*)

* **WARNING** * The below equations assume a constant acceleration, which is ***not*** true for a ship expending mass (for instance,

propellant). $Ai = F/Mc$ so as the ship's mass goes down, the acceleration goes up.

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When you have two out of three of *average velocity* (**Va**), *change in distance* (**S**) or *time* (**T**)

**Va** = S / T

**S** = Va * T

**T** = S / Va

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When you have two out of three of *acceleration* (**A**), *change in velocity* (**V**) or *time* (**T**)

**A** = V / T

**V** = A * T

**T** = V / A

================================================

When you have two out of three of *change in distance* (**S**), *acceleration* (**A**), or *time* (**T**)

plus *Initial Velocity* (**Vi**)   *Note: if deaccelerating, acceleration* **A** *is* <u>*negative*</u>

**S** = (Vi * T) + ((A * (T^2)) / 2)

**A** = (S – (Vi * T)) / ((T^2) / 2)

**T** =  (**sqrt[**(Vi^2) + (2 * A * S)**]** – Vi) / A

If Vi = 0 then

**S** = (A * (T^2)) / 2

**A** = (2 * S) / (T^2)

**T** = **sqrt[**(2 * S) / A**]**

================================================

When you have two out of three of *change in distance* (**S**), *acceleration* (**A**), or *final velocity* (**Vf**)

plus *Initial Velocity* (**Vi**)   *Note: if* **Vf < Vi,** *then* **A** *will be negative (deacceleration)*

**S** = (Vf^2 – Vi^2) / (2 * A)

**A** = (Vf^2 – Vi^2) / (2 * S)

**Vf** = **sqrt[**Vi^2 + (2 * A * S)**]**

If Vi = 0 then

**S** = (Vf^2) / (2 * A)

**A** = (Vf^2) / (2 * A)

**Vf** = **sqrt[**2 * A * S**]**

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If the ship constantly accelerates to the midpoint, then deaccelerates to arrive with zero velocity at the destination:

**T** = 2 * **sqrt[**S / A**]**

**S** = (A * (T^2)) / 4

**A** = (4 * S) / (T^2)

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**THRUST (Newtons or kg mt/sec)**

**F** = Mbs * A
= Mdot * Ve
= Mdot * g * Isp
= (dMp * Ve) / dT

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**THRUST POWER (kW)**

**Pw** = (Mdot * (Ve^2)) / 2
**Pw** = (dMp * (Ve^2)) / (2 * dT)

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**SPECIFIC IMPULSE (seconds)**

**Isp** = Ve / g
= F / (g * Mdot)

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**PROPELLANT MASS FLOW (kg/sec)**

**Mdot** = dMp / dT
= F / (g * Isp)
= F / Ve

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**VELOCITY OF EXHAUST (m/sec)**

**Ve** = g * Isp
= F / Mdot
**Ve/c** = **sqrt[** epsilon * (2—epsilon)**]**
*Ve/c = exhaust velocity in fractions of the velocity of light*

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**MASS OF PROPELLANT BURNT IN CURRENT BURN (kg)**

**dMp** = Mdot * dT
= (F * dT) / (g * Isp)
= (F * dT) / Ve

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**SPECIFIC POWER (kW/kg)**

**alpha** = Pw / Mps

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**CHARACTERISTIC VELOCITY**

**Vch** = **sqrt[** 2 * alpha * dT **]**

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**SHIP'S TOTAL MASS (kg)**

**Mt** = Mp + Mpl + Mps + Ms

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**SHIP'S MASS EMPTY (all propellant burnt) (kg)**

**Me** = Mt — Mp

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**SHIP'S MASS AT END OF BURN (kg)**

**Mbe** = Mbs — Mbp

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**SHIP'S MASS RATIO (dimensionless number)**
**Lambda** = Mt / Me
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**SHIP'S TOTAL VELOCITY CHANGE CAPABILITY (m/sec)**

**deltaV** = Ve * **1n[**Lambda**]**

= g * Isp * **1n[**Lambda**]**

*relativistic rocket formula*

**deltaV/c** = ( λ**^[**(2*Ve) /c**]** –1) / (λ**^[**(2*Ve) /c**]**+1)

**deltaV/c** = ( λ**^[**2*(**sqrt[**ε*(2–ε)**]**)**]** –1) / (λ**^[**2*(**sqrt[**ε*(2–ε)**]**)**]**+1)

*deltaV/c = vehicle final velocity expressed as a fraction of the velocity of light*
==================================================
**MAXIMUM DURATION OF BURN (seconds)**
**dTm** = Mp / Mdot
==================================================
**VELOCITY CHANGE OF CURRENT BURN (m/sec)**

**deltaVb** = Ve * **1n[**Mbs / Mbe**]**
==================================================
**ACCELERATION (m/sec^2)**

**A** = F / Mc

= (Mdot * Ve) / Mc

= (Mdot * g * Isp) / Mc

**Random sample of ship propulsion specifications**

Some solid fuel rockets have Lambda = 20 to 60.
Liquid fuel chemical rockets have a maximum Lambda of 12.
For a multi—stage rocket, the mass ratio is the
product of each stage's mass ratio.
A primitive value for alpha = 0.1 kW/kg.
In the near future alpha will equal 0.3 kW/kg.

**CHEMICAL ROCKET**

| **Propellant** | **Isp** | |
|---|---|---|
| Hydrogen—Fluorine ($F_2/H_2$) *ideal* | 528 | |
| Hydrogen—Oxygen ($O_2/H_2$) *space shuttle* | 460 | |
| Hydrogen—Oxygen ($O_2/H_2$) *ideal* | 528 | Epsilon = 1.5e—10 |
| ($O_3/H_2$) *ideal* | 607 | |
| ($F_2$/Li—$H_2$) | 703 | |
| ($O_2$/Be—$H_2$) | 705 | |
| Free Radicals (H+H)à$H_2$ | 2,130 | |
| Metastable Atoms (e.g. Helium) | 3,150 | |

**SATURN V FIRST STAGE**
**Isp** = 430 seconds
**F** = 3.41e7 newtons

**SPACE SHUTTLE**
**Isp** = 455 seconds
**F** = 2.944e7 newtons
**Mt** = 1.99e6 kg

**NERVA/DUMBO "Atomic Rocket"**
**Isp** = 850 to 2950 seconds
**F** = 1e5 to 1.3e6 newtons

**ORION "old Bang—bang"**
**Isp** = 1000 to 5000 seconds, second generation = 1e4 to 2e4

**ARCJET (Electrothermal)**
**Isp** = 800 to 1200 seconds

**MPD (Electromagnetic)**
**Isp** = 2000 to 5000 seconds

**ION (Electrostatic)**
**Isp** = (1/g) ∗ **sqrt[** 2 ∗ (q/m) ∗ Va**]**
q = charge of individual ion
m = mass of individual ion
Va = voltage or potiential difference through which ions are accelerated

**Isp** = 5e3 to 4e5 seconds
**F** = 4.6e—4 to 100 newtons (pretty pathetic, eh?)
*John Schilling <schillin@spock.usc.edu>*: A typical ion engine operating at a specific impulse of ~2500 seconds, consumes 25 kW of power per newton of thrust. This assumes 50%

overall efficiency; even an unattainable 100% would still leave you with over 10 kW/N. High molecular weight is a **good** thing for ion thrusters, which is precisely why people are looking at C60 (buckminsterfullerenes) for the application. And why contemporary designs use Xenon, despite its cost of ~$2000/kg.

Low molecular weights are good for thermal rocket, because exhaust velocity is essentially the directed thermal velocity of the gas molecules, the thermal velocity is proportional to the square root of temperature over molecular weight, and there is a finite upper limit to temperature.

With ion thrusters, thermal velocity is irrelevant. Exhaust velocity comes from electrostatic acceleration and is proportional to the square root of grid voltage over molecular mass. There is no real upper limit to grid voltage, as a few extra turns on the transformer are cheap and simple enough.

However, you have to ionize the propellant before you can accelerate it, and the ionization energy is pure loss. This wasted ionization energy has to be payed for each and every particle in the exhaust, so the fewer individual particles you have to deal with, the less waste. The heavier the particles, the fewer you need for any given mass flow rate.

Also helps if the ionization energy of the particles is low, of course, but there's much more variation in atomic and molecular weight than in first ionization energy.

*Frank Crary:* The ions will be accelerated out through the grid at a velocity,

v = **sqrt[**2 * q * V/m**]**

where q is the charge of the ions, V the voltage applied and m the mass of the ions. This will produce a force

F = m * r * v = r * **sqrt[**2 * q * V / m**]**

where r is the rate at which particles are ionized and accelerated,
in particles per second. At the same time, the ion beam produces a current

I = q * r

and a current flowing across a voltage, V, requires input power

P = I * V = 0.5 * m * (v^2) * r = 0.5 * v * F

The last form of that is a fundamental limit on ion drives: For a given power supply (i.e. a given mass of solar panels, nuclear reactor or whatever), you can get a high exhaust velocity or a high thrust, but not both. Unfortunately, both is exactly what you want: The fuel requirements for a given maneuver depend on the exhaust velocity.

**LIQUID CORE NUCLEAR REACTOR**
**Isp** = 1300 to 1600 seconds
**epsilon** = 7.9e–4

**GASEOUS CORE NUCLEAR REACTOR**
**Isp** = 3570 to 7000 seconds
**Ve** = 3.5e4 to 5e4 m/sec
**F** = 3.5e6 to 5e6 newtons
**Mps** = 5e4 to 2.55 kg
**epsilon** = 7.9e–4

**GASEOUS CORE COAXIAL FLOW REACTOR**
**Isp** = 1800 seconds
**Ve** = 17,640 m/sec
**F** = 1.78e7 newtons
**Mps** = 1.27e5 kg
**epsilon** = 7.9e—4

**DAEDALUS (Fusion Microexplosions)**
**Isp** = 1e6
**epsilon** = 4e—3

**BORON FUSION**
$^{11}B_5$ + p à 3( $^{4}He_2$ ) + 16Mev

that is, bombard Boron-11 with protons. A complicated reaction ends with helium and no pesky nuclear particles. 16 million electon volts gives an exhaust velocity of better than 10,000 km/sec, which translates into a theoretical specific impulse of something over a million seconds.

What's the catch?

*schillin@spock.usc.edu (John Schilling)*

The catch is, you have to arrange for the protons to impact with 300 keV of energy, and even then the reaction cross section is fairly small. Shoot a 300 keV proton beam through a cloud of boron plasma, and most of the protons will just shoot right through. 300 keV proton beam against solid boron, and most will be stopped by successive collisions without reacting. Either way, you won't likely get enough energy from the few which fuse to pay for accelerating all the ones which didn't.

Now, a dense p-B plasma at a temperature of 300 keV is another matter. With everything bouncing around at about the right energy, sooner or later everything will fuse. But containing such a dense, hot plasma for any reasonable length of time, is well beyond the current state of the art. We're still working on 25 keV plasmas for D-T fusion.

If you could make it work with reasonable efficiency, you'd get on the order of ten gigawatt-hours of usable power per kilogram of fuel.

*Paul Dietz <paul@interaccess.com>*

Unfortuantely, this discussion ignores side reactions:

p + $^{11}B_5$ à $^{12}C$ + $\gamma$

$^{4}He$ + $^{11}B_5$ à $^{14}N$ + n

The first is quite a bit less likely than the à3( $^{4}He_2$ ) reaction, but the photon is very energetic and penetrating. The second reaction there occurs with secondary alpha particles before they are thermalized.

**HYPOTHETICAL FUSION TORCH**
**Isp** = 5e4 to 1e6 seconds
thrust/mass ratio 1e—4 to 1e—5
**epsilon** = 4e—3
*Erik Max Francis:* For fusion reactions, the yield is about 6.3e14 J/kg, which gives us an maximum exhaust velocity for fusion drives of about 3.6e7 m/s

**ANTIMATTER**
**Isp** about 3.06e7 seconds
**epsilon** = 1.0

**PHOTON DRIVE**
**Isp** about 3.06e7 seconds
*Erik Max Francis <max@alcyone.com>*
even using an ideal drive (exhaust velocity = c), the mass ratio you'd need to have a deltavee of 0.995 c would be 2.12e4.  That is, you'd need 21,200 times more fuel than payload.
The mometum of a photon is given by
  p = E/c,
where E is the energy of the photon, and so the thrust delivered by a stream of them is
  dp/dt = dE/dt/c
or
  F = P/c
where F is the thrust and P is the power.  To get a thrust of 1 N, you need a power of 300 MW.  Yes, three hundred megawatts!

**IBS (Interplanetary Boost Ship)** ***Agamemnon*** (from Jerry Pournelle's "Tinker")
Fusion powered ion drive
**Mt** = 1e8 kg
**Mp** = 7.2e7 kg
**Mpl** = 2e7 kg
**Lambda** = 2.57
**Ve** = 2e5 m/sec
**Isp** = 20,408 seconds
**deltaV** = 2.6e5 m/sec
**F** = 5.6e6 newtons
**Mdot** = 28 kg/sec

Typical mission: Earth—Pallas, with Pallas at 4 AU from Earth.
Boosts at 1/100g for about 15 days to 140km/sec.
Coast for 40 days. Deaccelerate for another 15 days.

# OTHER USEFUL EQUATIONS

## CIRCULAR ORBITAL VELOCITY

$\mathbf{V_c} = \mathbf{sqrt(}\ (G*M_*)/R_o\mathbf{)}$
$V_c$ = circular orbital velocity
G = universal gravitational constant = 6.67206e–11 $Nm^2/kg^2$
$M_*$= mass of the star (sun = 1.99e30 kg)
$R_o$= distance from center of the star

## STELLAR ESCAPE VELOCITY

$\mathbf{V_e} = V_c * \mathbf{sqrt(}2\mathbf{)}$

## HYPERBOLIC EXCESS VELOCITY

$\mathbf{V_\infty} = V - V_e$
Once beyond the influence of the star, a ship will cruise indefinately at the hyperbolic excess velocity in aproximately a straight line.

## RELATIVISITIC MOTION

$\gamma = 1 / \mathbf{sqrt\ (}\ 1-((v\hat{\ }2) / (c\hat{\ }2))\mathbf{)}$

For two inertial (unaccelerated) frames of reference, if frame S' is moving with respect to frame S with velocity V, in the postive direction along the x–axis during time t, then:
$\mathbf{x'} = \gamma * (x - (V * t))$
$\mathbf{x} = \gamma * (x' + (V * t))$
$\mathbf{t'} = \gamma * (t - ((V * x)/(c\hat{\ }2)))$
$\mathbf{t} = \gamma * (t' + ((V * x')/(c\hat{\ }2)))$

For a rocket moving with *constant acceleration* **a**, due to thrusting in its *proper frame*, then the total elapsed proper time Δt' (as time is measured on the rocket) over distance S:
$\mathbf{\Delta t'} = (c/a) * \mathbf{cosh^{-1}(}1 + (a * S)/(c\hat{\ }2)\mathbf{)}$
where $\mathbf{cosh^{-1}}(x)$ = inverse hyperbolic cosine of x
As (a*S)/(c^2) approaches 1.0, the equation becomes
$\mathbf{\Delta t'} = (c/a) * \mathbf{1n(}(2 * a * S)/(c\hat{\ }2)\mathbf{)}$
The vehicle's velocity after accelerating for Δt' and reaching distance S is:
$\mathbf{V} = c * \mathbf{sqrt(}1 - (1 / (1 + ((a * S) / (c\hat{\ }2)))\hat{\ }2)\mathbf{)}$

If the rocket accelerates at **a** up to the midpoint, then deaccelerates at –**a** to destination:
$\mathbf{\Delta t'}= ((2 * c)/a)\mathbf{cosh^{-1}(}1 + (a/(2 * c\hat{\ }2)) * S\ \mathbf{]}$
As (a*S)/(c^2) approaches 1.0, the equation becomes
$\mathbf{\Delta t'} =((2 * c)/a)\mathbf{ln[}\ (a/(c\hat{\ }2))S\ \mathbf{]}$
Velocity at turnover is
$Vturnover = c*\mathbf{sqrt[}\ 1 - (\ 1 + (a/(2 * c\hat{\ }2)) * S\ )\hat{\ }(-2)\ \mathbf{]}$

*Erik Max Francis <max@alcyone.com>*

**Δ**v = u **1n** λ/**sqrt[**1 + (u^2/c^2) **1n**^2 λ**]**

where **Δ**v is the deltavee, and λ is the mass ratio, or the ratio of the
initial to the final mass.

v = c t/**sqrt[**c^2/a'^2 + t^2**]**
r = c [(c^2/a'^2 + t^2) — c/a']
t' = (c/a') **ln [**(1 + a'^2 t^2/c^2)^(1/2) + a' t/c**]**
t = c v/a'/**sqrt(**c^2 — v^2**)**

a' = *subjective* acceleration
v = *objective* velocity
r = *objective* displacement
t = *objective* elapsed time
t' = *subjective* elapsed time
*"objective"* = from the rest frame (at rest relative to the departure point)
*"subjective"* = from the ship frame

Subjective (not objective) acceleration is constant; acceleration is all in one direction only.

*Example:*
t = c v/a'/**sqrt(**c^2 - v^2**)**
if v = 0.995 c and a' = 5000 gee
then t = 8.61e4 sec = 23.9 hours
Now plug t into
t' = (c/a') **ln [**(1 + a'^2 t^2/c^2)^(1/2) + a' t/c**]**
and get 2.04e4 sec = 5.67 hours
Plug t into
r = c [(c^2/a'^2 + t^2) - c/a']
to get objective displacement of 2.4e13 m (about 160 au)
*Bill Woods <wwoods@ix.netcom.com>:*
Assuming a magical stardrive which allows you to accelerate continuously at constant acceleration a, as measured onboard the ship,

a  : ship acceleration
tau: ship time (proper time)
d  : ship distance

T: Earth time
D: Earth distance
A: Earth acceleration

Mo: initial mass
M : mass of ship

theta(tau) = (a/c)tau        : velocity parameter
beta = v/c  = tanh(theta)
          = tanh((a/c)tau)
gamma = 1/sqrt[ 1 - beta^2 ]

v(tau) = c*tanh[(a/c)tau]
D(tau) = (c^2/a)*( cosh[(a/c)tau] - 1 )

tau(D) = (c/a)arccosh[ (a/c^2)D + 1 ]
d(tau) = D/cosh(theta) = (c^2/a)*( 1 - sech[(a/c)tau] ) -> c^2/a
d ~ c^2/a for tau > 6c/a

T(tau) = (c/a)sinh((a/c)tau)      (a/c)T  =   sinh( (a/c)tau )
tau(T) = (c/a)arcsinh((a/c)T)      (a/c)tau = arcsinh( (a/c)T  )

Alternately, in the frame of a stationary observer,
your acceleration is measured as:

A = a / gamma^3

A(v) = a*sqrt( 1 - (v/c)^2 )^3
D(T) = (c^2/a)*( sqrt[1 + ((a/c)T)^2] - 1 )
T(D) = (c/a)sqrt[ ( (a/c^2)D + 1 )^2 - 1 ]
v(T) = a*T / sqrt[ 1 + ((a/c)T)^2 ]
= c / sqrt[ 1 + (c/aT)^2 ]
beta(T) = v(T)/c = 1 / sqrt[ 1 + (c/aT)^2 ]

tau(T) = (c/a)ln[ (a/c)T + sqrt( 1 + ((a/c)T)^2 ) ]

A(T) = a / sqrt( 1 + ((a/c)T)^2 )^3

For acceleration at 10 m/s^2, the time taken to reach various distances
is:

| Earth Dist : | Earth time | speed | ship time | ship distance |
|---|---|---|---|---|
| .06 ly : | 0.34 yr | 0.34 c | 0.34 yr | 0.06 ly |
| 0.25 ly : | 0.73 yr | 0.61 c | 0.67 yr | 0.20 ly |
| 0.50 ly : | 1.10 yr | 0.755 c | 0.94 yr | 0.33 ly |
| 1 ly : | 1.70 yr | 0.873 c | 1.28 yr | 0.49 ly |
| 2 ly : | 2.79 yr | 0.9467 c | 1.71 yr | 0.64 ly |
| 4 ly : | 4.86 yr | 0.9814 c | 2.22 yr | 0.77 ly |
| 10 ly : | 10.91 yr | 0.99622 c | 2.98 yr | 0.87 ly |
| 25 ly : | 25.93 yr | 0.99932 c | 3.80 yr | 0.92 ly |
| 50 ly : | 50.94 yr | 0.99982 c | 4.44 yr | 0.93 ly |
| 100 ly : | 100.95 yr | 0.999947 c | 5.09 yr | 0.94 ly |
| 1000 ly : | 1000.95 yr | 0.999991 c | 7.27 yr | 0.95 ly |
| 10000 ly : | 10000.98 yr | 0.999992 c | 9.46 yr | 0.95 ly |
| | | | | d -> 0.9500 ly |

For a trip which goes from standing start to standing finish,
calculate the time to cover half the distance,
then double the T and tau variables.

DistAlphaCen   = 4.3 ly = 41 Pm = 41e15 m
1/2 DAC    = 20.5e15 m
1/2 tauAC   = 55.7e6 sec
1/2 TAC    = 93.7e6 sec
TauToAlphCen   = 111e6 sec = 3.5 yr
TimeToAlphaCen  = 187e6 sec = 5.9 yr

For a perfectly efficient photon rocket,

theta = ln(Mo/M) , so        M(tau) = Mo*e^[-(a/c)tau]

or more conveniently,        theta(Tau1/2) = ln(2) = 0.7

so the rocket's halflife is  Tau1/2 = 0.7c/a

for instance, for a = 1 kgal (= 1000 cm/s^2 ~ 1 "gee")
Tau1/2 = 21e6 s ~ 8 months

TauToAlphaCen = 111e6 s = 3.5 years ~ 5.3 Tau1/2
so initially the rocket must be more than 31/32 fuel.

**LASER LIGHTSAIL**

```
                         /#        ^
                      /   #        |
                   /      #        |
     | d        /         #  sail  |
     v   /theta           #        |
+-----+/                   #        |
|laser|<-----r-------->#       ds
+-----+\                   #        |
     ^    \               #        |
     |       \            #        |
                 \        #        |
                    \     #        |
                       \#        v
```

**$\theta$** = diffraction limited beam divergence angle
**r** = separation between laser and sail
**$d_s$** = sail diameter
**d** = laser transmitter aperture
**$\lambda$** = radiation wavelength

by Rayleigh's Criteria:
$\sin\theta \approx \theta = (1.22*\lambda) / d$

from the geometry of the figure:
$\sin\theta \approx \theta = d_s / (2*r)$

Therefore:
$d_s / (2*r) = (1.22*\lambda) / d$

The distance at which the beam would just fill the sail is:
$r = (d_s*d) / (2.44*\lambda)$

the energy of a photon is
$E = (h*\nu)$
h = Planck's constant = 6.6260755e–35 J/Hz
$\nu$ = photon's frequency

$\lambda = h / p$
$\lambda$ = photon's wavelength
p = photon's momentum

$c = (\nu*\lambda)$
c = speed of light in vacuum

therefore:
$p = E/c$

If a beam of total photon energy $E_b$ is completely *absorbed* by a sail (inelastic collision), the momentum lost by the beam and gained by the sail is

$\Delta \mathbf{p}_i = E_b / c$

if the sail is 100% reflective (elastic collision), the momentum is

$\Delta \mathbf{p}_e = (2*E_b) / c$

The starship's momentum change per unit time is

$\mathbf{p}'_e = M_s * V'_s$

$M_s$ = starship mass

$V'_s$ = starship acceleration

The starship's acceleration is

$\mathbf{V}'_s = (2*E_b) / (M_s*c)$

**BUSSARD RAMJET**

$M_s$ = mass of ramjet starship
$V_s$ = velocity
$\rho$ = ion density of interstellar medium
A = effective intake area of ramscoop
$m_i$ = average mass of scooped-up interstellar ions
$\varepsilon$ = fraction of reaction mass converted into exhaust kinetic energy by reactor
$V'_s$ = ship acceleration
$M'_f$ = fuel mass collected per second
$V_e$ = exhaust velocity relative to interstellar medium

$M_s * V'_s = M'_f * V_e$
$\mathbf{M_f} = A * \rho * m_i * V_s$
$\mathbf{V_e} = \varepsilon * ( (c^2) / V_s)$
$\mathbf{V'_s} = A * \rho * m_i * \varepsilon * ( (c^2) / M_s)$

Note that ramjet acceleration is independent of spacecraft velocity!
*Example:*
A = 3.14e12 m *( scoop diameter of 2000 km)*
$M_s$ = 1e6 kg
$\varepsilon$ = 1e–3
$\rho$ = 1e–6/$m^3$
$m_i$ = 1.67e–27 kg*(protons)*
therefore:
$V'_s$ = 0.5 m/$sec^2$ = 0.05 g *(note: this does not compute...)*

**Ionizing interstellar hydrogen by laser beam**
vol = volume traversed by a laser photon
L = beam length
$\lambda$ = wavelenght = 0.0916$\mu$m = 9.16e–8 m
**vol** = $\pi$ L $\lambda$^2/ 4
**vol** = $\pi$ * L * (($\lambda$^2) / 4) or ($\pi$ * L * ($\lambda$^2)) / 4

tVol = total volume of entire laser beam
R = beam radius
**tVol** = $\pi$ * (R^2) * L

E = photon energy
h = Planck's constant = 6.6260755e–35 J/Hz
**E** = h c / $\lambda$
**E** = h * (c / $\lambda$) or (h * c) / $\lambda$

$E_i$ = laser energy for 100% ionization
$E_i$ = 4hcR^2/$\lambda$^2
$E_i$ = 4 * h * c * ((R^2) / ($\lambda$^2)) or (4 * h * c * (R^2)) / ($\lambda$^2)
*Example:* if R = 50,000 km then $E_i$ = 2e12 joules
If laser is turned on for 50 days and the pulse is repeated every 230 days, the laser power is 5e5 watt. Because light travels 1.3e12 km in 50 days,

the necessary beam dispersion is 3e-8 radian.

**Fusion Reactions**

proton-proton

| | |
|---|---|
| $^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \nu$ | 2 protons yield a deuteron, positron, and neutrino |
| $e^{+} + e^{-} \rightarrow \gamma$ | positron + electron yield a gamma ray |
| $^{1}H + {}^{2}H \rightarrow {}^{3}He + \gamma$ | proton + deuteron yield a $^{3}He$ nucleus + gamma |
| $^{3}He + {}^{3}He \rightarrow {}^{4}He + 2\ {}^{1}H$ | 2 $^{3}He$ yield an alpha particle and 2 protons |

0.007 of initial reactant mass is converted into energy.
Neglecting the energy of the neutrino, 26.20 Mev is released.
Due to low cross-section, the proton-proton reaction is exceedingly difficult to initiate.

## LOSING SHIP'S ATMOSPHERE THROUGH A HULL BREACH

v = **sqrt(** 2 * P / rho **)**
v = effective speed of the air as it passes through the hole
(ignoring friction)
P = difference between inside and outside pressures
rho = mass density of the air.
Assuming Earth-normal pressure and density inside, and zero pressure outside, the effective speed works out to a smidgen under 400 m/sec.

dm/dt = A * **sqrt(** 2 * P * rho **)**
dm/dt = the rate (mass per unit time) at which air leaks into vacuum,
A = Area of the hole it's leaking through
P = Pressure inside the room far from the hole
rho = density inside the room far from the hole

## GAMMA RAY BURSTERS

The energy released by a gamma ray bursters is about 10^45 J (that's less energy released than in a type II supernova, by the way, albeit most of a supernova's energy is released as neutrinos) and is released over a few seconds. Since the duration is so short, we can treat it as being instantaneous (you're unlikely to reach safety in two seconds), and so we will deal in energy units rather than power units.

The energy is released almost entirely in gammas (hence the name). From this we can calculate the distance at which an unprotected human (suppose exposed surface area 1 m^2, total mass 70 kg) would reach a lethal dose (LD50 is 4 Sv = 400 rem).

So the question is, at which distance does the dose equivalent reach 4 Sv? The weighting factor for gammas is 1, so 4 Sv corresponds to 4 Gy, which is 4 J/kg of ionizing radiation deposited. Since our healthy human has a mass of 70 kg, this corresponds to 280 J. Further, the surface area of 1 m^2 leads us to a lethal intensity of 280 J/m^2.

So the question becomes: What distance does our patient have to be from the gamma ray burster to experience a burst intensity of 280 J/m^2? Intensity, source energy, and distance are related by

I = E/(4 * π * (R^2))

or solving for R,

R = **sqrt[**E/(4 * π * I)**]**

and solving for R, we get about 5e20 m. Note, this is about 50 kly.

This doesn't take into account attenuation due to dust, which will of course significantly reduce the flux. However, the point here is that, we're talking about a _lot_ of radiation. Enough to sterilize a good portion of a galaxy.

## STELLAR POPULATION DENSITY

The solar local density of stars is about 0.13 per pc^3. For 50 y, the transmissions have travelled 50 ly, or about 15 pc. The volume encompassed in a sphere of that radius is about 15 000 pc^3, and so the number of stars contained in that sphere should be about 2000.

## DESTROYING A PLANET

The gravitational potential energy of a self-gravitating, uniform sphere is

$U = (3/5)\, G\, M^2 / R$

$G = 6.67e{-11}$ and for the Earth $R = 6.378e{+6}$ and $M = 5.98e{+24}$ (all units in MKS), implying a potential energy of 2.258e+32 joules. You need to supply at least this much energy to "reduce the Earth to gravel" *and* remove all the pieces to infinity. This amount of energy is equal to the total conversion to energy of 2.51e+15 kg of matter. That would take at least 1.25e+15 kg of antimatter, assuming total conversion (an assumption not valid for a surface blast; nor would all this energy be "useful" in destroying the Earth (most would be wasted blasting a very small percentage of the mass outward at much more than escape velocity)).

*Eric Max Francis:* Total energy required to completely gravitational disrupt a uniform, spherical body of mass M and radius R:

$E = (9/15)\, G\, M^2/R$

*Example:* to destroy the Earth

$Me = 7E24$ kg
$Re = 6.75E6$ m
$G = 6.67E{-11}\ m^3/kg\text{-}s^2$

$E = 2.9E32$ Joules

In terms of the Sun's energy output:

$Ls = 3.7E26$ Watts
$2.9E32 / 3.7E26 = 7.8E5$ seconds = about a week

Or a five- to ten-mile chunk of antimatter, or a fifty-mile wide fusion bomb.
Note: $9/15 = 3/5 = 0.6$

## CLOSE APPROACH TO A NEUTRON STAR

*Erik Max Francis <max@alcyone.com>*

I looked at this a little more and this is the differential equation I came up with:

$$d^2 \phi/(dx\, dt) = \frac{6 G M (h \sin \phi - x \cos \phi)}{v\, l\, (x^2 + h^2)^{(1/2)}\, [(x^2 + h^2)^{(1/2)} - l]^2}$$

x(t) is the position along the "orbit" (idealized as a straight line); x = 0 (at t = 0) corresponds to the point of closest approach. v is dx/dt (taken as constant). h is the closest approach to the neutron star; 2 l is the length of the ship. M is the mass of the neutron star. phi(t) is the position angle of the ship, such that phi(t = 0) = pi/2. (dphi/dt)(t = 0) = v/h.

## "PRESSOR" DRIVE

*Erik Max Francis <max@alcyone.com>*

In past conversations, the idea for a "pressor drive" has been brought up – a reactionless drive which gets around by "pushing" off of masses. One can handwave that it accomplishes this by obeying conservation laws; e.g., the ship experiences a force by "pushing" on, say, a planet, but the planet experiences an equal and opposite force.

So let's analyze some possible characteristics of the drive, and backengineer what quantitative characteristics it must have to accomplish goals.

So we'll start with this: The drive "presses" on nearby masses when turned on. (It can be fine-tuned, so for a given mass at a given distance, there is a maximum force that the drive can experience, but it can be "throttled down" so it doesn't _have_ to be at the maximum.) To make things interesting, let's say that the drive operates by "reflecting" something off of the masses in question, so that the force applied to the ship varies as the inverse fourth power of the distance. If it is also directly proportional to the mass, then the equation for the thrust F achieved from one mass M and a distance r is

$$F = k M/r^4,$$

where k is the constant of proportionality, with (SI) units of N m^4/kg, or m^5/s^2.

Now let's start with the backengineering. Let's say we want to use this drive for interstellar (slower than light) travel. Let's calculate the straight-line deltavee achieved by turning one of these drives on in the vicinity of a mass.

Acceleration a is related to thrust F by

$$a = F/m,$$

where m is the mass of the ship. Substituting our equation above, we get the equation of motion for the drive:

$$a = k (M/m)/r^4.$$

Since k, M, and m are all constants, we can define another constant K == k M/m and simply the equation of motion:

$$a = K/r^4.$$

a = dv/dt. We're interested in deltavee accumulated by the ship being accelerated by the drive as it moves through a certain distance, so we want our integration variables to be v and r, not v, r, and t. So we can do some mathematical manipulation:

$$a = dv/dt = (dv/dr) (dr/dt) = v\, dv/dr,$$

and now we can write

$$v\, dv = K r^{-4}\, dr.$$

Integrating v from 0 (at rest) to v (deltavee), and r from r_o (initial distance) to r (final distance), and we get

$$(1/2) v^2 = (1/3) K (1/r_o^3 - 1/r^3).$$

And solving for v,

$$v = [(2/3)\, K\, (1/r_o^3 - 1/r^3)]^{(1/2)}.$$

We can resubstitute $K$ with $k M/m$, or a more convenient form, $k \rho$, where $\rho \equiv M/m$, the ratio of the mass of the pressed body to the mass of the pressing body:

$$v = [(2/3)\, k\, \rho\, (1/r_o^3 - 1/r^3)]^{(1/2)}.$$

Now the question is: What choice of $k$ will make this drive suitable for interstellar travel? We can backengineer this problem by selecting for a situation where the drive will get us to relativistic velocities. Let's say that we start from Earth orbit ($r_o = 1.5 \times 10^{11}$ m), accelerates out to infinity ($r \to \infty$), and the deltavee is around lightspeed ($v = c$)*. If we choose the mass of the ship such that's comparable to a large naval vessel ($\sim 10^8$ kg) and it's pressing against the Sun ($\sim 10^{30}$ kg), then $\rho \sim 10^{22}$.

You can solve for $k$ and find

$$k = v^2/[(2/3)\, \rho\, (1/r_o^3 - 1/r^3)].$$

and with these parameters, $k \sim 10^{28}$ N m^4/kg. So with this parameter, the drive can be used for interstellar travel (though one couldn't drive it at full throttle, for the initial acceleration would be ~10 000 gee!).

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* This is an effective way to start with the qualitative behaviors you're interested in, and then refine the quantitative behaviors to get the specific applications of a device.

** Clearly the ship will not reach c, but we're just using this as a potential for acceleration, not as an actual speed reached. Besides, the computations performed weren't relativistic anyway.